The Unpleasant Monetarist Arithmetic

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Sargent and Wallace (1981) unpleasant monetarist arithmetic:

"Some unpleasant monetarist arithmetic," Federal Reserve Bank of Minneapolis *Quarterly Review*, 1-17, Fall.

Main idea:

Although the monetary authority keeps inflation low in the present, if the fiscal authority sets the budget independently, then the monetary authority will be forced to create money and tolerate more inflation in the future.

- The fiscal authority autonomously sets the primary budget balance;

- The monetary authority starts by trying to control the money supply but ends up having to choose between more inflation in the present or more inflation in the future;

- If the monetary authority chooses a low value for the money supply in the current period, controls the price level, keeping it low;

- This contributes to the increase of the real value of the stock of government debt;

- In the future, the monetary authority has to resort to seigniorage to ensure consistency between the price level and the government budget constraint;

- In practice, there is a “chicken game” between the two authorities, who try to implement simultaneously an active monetary policy and an active fiscal policy;

- The unpleasant monetarist arithmetic is then the result of the “forced”/passive reaction of the monetary authority to the active fiscal policy.

- Consolidated government budget constraint (Treasury plus Monetary Authority)

$$D_t = \frac{H_t - H_{t-1}}{P_t} + B_t - B_{t-1}(1 + R_{t-1}) \qquad (1)$$

D – real primary government spending minus real government revenues;
H – monetary base;
B – one period government debt;
P – price level;
R – real interest rate;

- Per capita government budget constraint, N – population,

$$\frac{D_t}{N_t} = \frac{H_t - H_{t-1}}{N_t P_t} + \frac{B_t}{N_t} - \frac{B_{t-1}}{N_t}(1+R_{t-1}) \qquad (2)$$

$$\frac{B_t}{N_t} = \frac{B_{t-1}}{N_t}(1+R_{t-1}) + \frac{D_t}{N_t} - \frac{H_t - H_{t-1}}{N_t P_t} \qquad (3)$$

- Using a constant growth rate n for population (and real income),

$$N_t = (1+n)N_{t-1} \qquad (4)$$

after rearranging (3) we can write

$$\frac{B_t}{N_t} = \left(\frac{1+R_{t-1}}{1+n}\right)\frac{B_{t-1}}{N_{t-1}} + \frac{D_t}{N_t} - \frac{H_t - H_{t-1}}{N_t P_t} \qquad (5)$$

- Assuming a constant growth rate θ for the monetary base, during t=2, 3, …, T,

$$H_t = (1+\theta)H_{t-1} \qquad (6)$$

- For $t > T$, the path of H is determined by the condition that *Bt/Nt* remains constant at the level observed in $t=T$, at the per capita government debt stock b_T^{θ}

- The price level is proportional to *Ht/Nt* , with a positive constant h

$$P_t = (1/h)\frac{H_t}{N_t} \qquad (7)$$

- The inflation rate is given by

$$\frac{P_t}{P_{t-1}} = \frac{(1/h)}{(1/h)} \frac{H_t / N_t}{H_{t-1} / N_{t-1}} \quad (8)$$

$$\frac{P_t}{P_{t-1}} = \frac{(1+\theta)}{(1+n)} \quad (9)$$

To see how inflation before T impinges on inflation after T, substitute b_T^{θ} in (5) for $B_t/N_t = B_{t-1}/N_{t-1} = b_T^{\theta}$ (the per capita debt stock at T),

$$b_T^{\theta} = \left(\frac{1+R_{t-1}}{1+n}\right) b_T^{\theta} + \frac{D_t}{N_t} - \frac{H_t - H_{t-1}}{N_t P_t} \qquad (10)$$

and using (7) [$H_t = P_t N_t h$] and (4), [$N_t = (1+n) N_{t-1}$]

$$1 - \frac{P_{t-1}}{P_t}\frac{1}{1+n} = \left[\left(\frac{R_{t-1}-n}{1+n}\right) b_T^{\theta} + \frac{D_t}{N_t}\right] / h \qquad (11)$$

- With $(R_{t-1} - n) > 0$ (by assumption), the right-hand side of (11) is higher the higher is b_T^{θ}

- This implies a lower 2nd term in the left-hand side of (11), i.e. lower $(P_{t-1} / P_t)(1/(1+n))$ and higher inflation.

- Therefore, the higher the per capita stock of government debt at time T, the higher the inflation rate, π:

$$\Delta b_T^{\theta} \rightarrow \Delta \pi$$

- The smaller θ (growth rate of monetary base) the higher b_T^{θ} (per capita debt stock at *T*)

- Solve (5) for *bt*=*Bt*/*Nt*

$$\frac{B_1}{N_1} = \frac{(1+R_0)B_0}{N_1} + \frac{D_1}{N_1} - \frac{H_1 - H_0}{N_1 P_1} \qquad (12)$$

and using as the par value of debt issued in t=0, $B_0(1+R_0) = \bar{B}_0 / P_1$

$$\frac{B_1}{N_1} = \frac{\bar{B}_0}{N_1}\frac{1}{P_1} + \frac{D_1}{N_1} - \frac{H_1 - H_0}{N_1 P_1} \qquad (13)$$

- With (13) and (7) [price level proportionality to *H*] it is possible to solve for b_1 [which does not depend on θ].

- The per capita government budget constraint (5) can be written as

$$b_t = \left(\frac{1+R_{t-1}}{1+n}\right) b_{t-1} + \frac{D_t}{N_t} - \frac{H_t - H_{t-1}}{N_t P_t} \qquad (14)$$

- Using the growth rate of monetary base (6) and its proportionality to prices (7),

$$b_t = \left(\frac{1+R_{t-1}}{1+n}\right) b_{t-1} + \frac{D_t}{N_t} - \frac{h\theta}{1+\theta} \qquad (15)$$

This result can be generalised for t=2, 3, …, T:

$$b_T^{\theta} = \frac{\prod_{j=s}^{t-1}(1+R_j)}{(1+n)^{t-s}} b_1 + \sum_{s=2}^{t}\left[\frac{\prod_{j=s}^{t-1}(1+R_j)}{(1+n)^{t-s}} \frac{D_s}{N_s}\right] - \frac{h\theta}{1+\theta}\sum_{s=2}^{t}\left[\frac{\prod_{j=s}^{t-1}(1+R_j)}{(1+n)^{t-s}}\right] \qquad (16)$$

From (16), the smaller the growth rate of the monetary base, θ, the higher the per capita debt stock b_T^{θ}

$$\frac{\partial b_T^{\theta}}{\partial \theta} = -\frac{h}{(1+\theta)^2} \langle 0$$

Therefore, we have the unpleasant monetarist arithmetic

Lower growth rate of monetary base before T	$\rightarrow$	Higher debt accumulation in T	$\rightarrow$	Higher inflation after T

Implying that tighter money today means higher inflation eventually

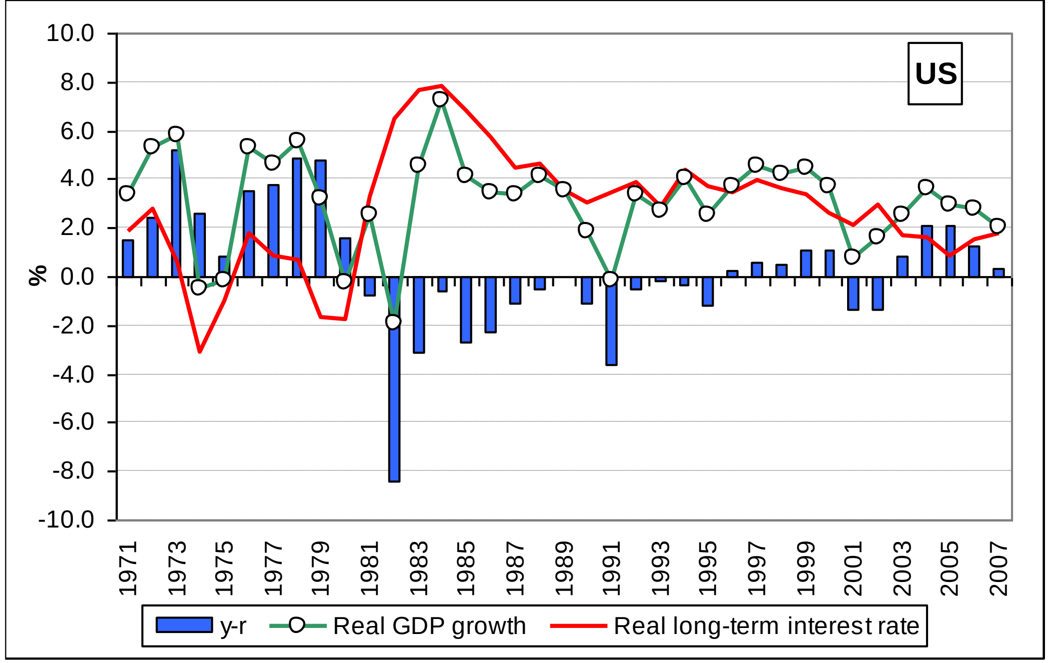
$$\nabla \theta \rightarrow \Delta b_T^{\theta} \rightarrow \Delta \pi$$

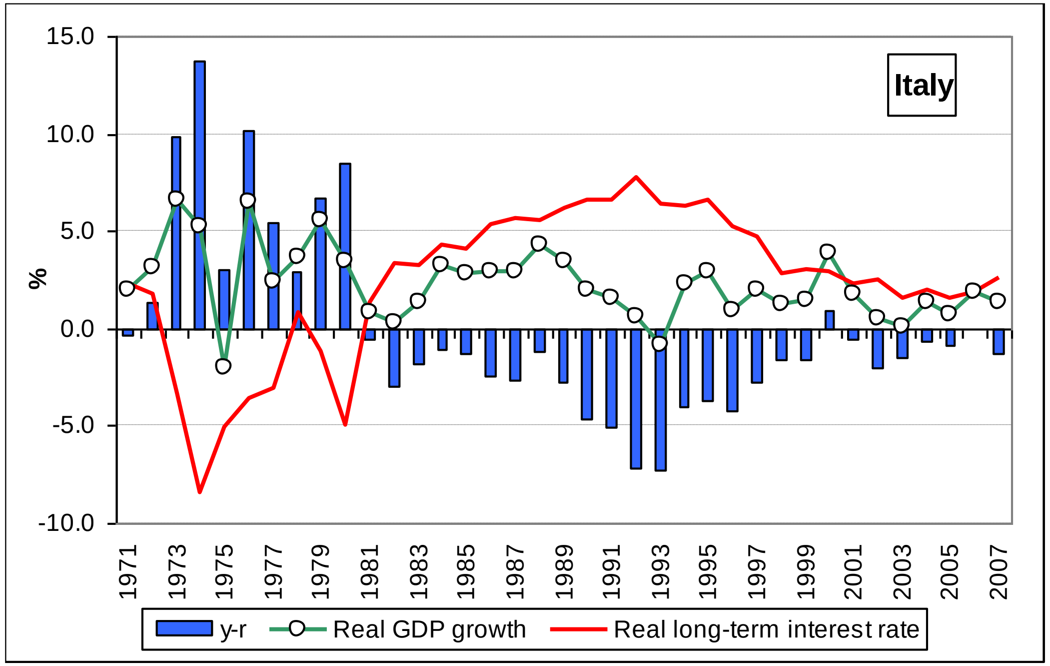
- Sargent, T. and Wallace, N. (1975). “'Rational expectations, the optimal monetary instrument and the optimal money supply rule”, *Journal of Political Economy*, 83 (2), 241-254.

- Sargent, T. and Wallace, N. (1981). “Some unpleasant monetarist arithmetic,” Federal Reserve Bank of Minneapolis *Quarterly Review*, 1-17, Fall.

  http://www.minneapolisfed.org/research/QR/QR531.pdf

Background material





Source: Eurostat, IMF, EC.